Journal of Engineering Mathematics, Vol. 8, No. 1, January 1974
Noordhoff International Publishing–Leyden
Printed in The Netherlands

Application of the sampling theorem to boundary value problems

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(Received February 8, 1973)

SUMMARY
The generalized sampling theorem is used to facilitate the solution of a conjugated boundary value problem of the Graetz type. The analysis is applied to determine the effects of axial conduction on the temperature field in a fluid in laminar flow in a tube. This represents the first application of the sampling theorem outside of the area of communications theory.

1. Introduction

The temperature field in the thermal entry region for Poiseuille flow in a heated circular tube is governed by the axisymmetrical thermal energy equation in cylindrical coordinates, which in dimensionless form is

$$(1-r^2)\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{\text{Pé}^2}\frac{\partial^2 T}{\partial x^2}, \tag{1}$$

where the Péclet number is defined by $\text{Pé}=u_m r_0/\alpha$, u_m is the maximum velocity in the tube, r_0 is the tube radius and α is the thermal diffusivity. The dimensionless radial and axial coordinates, x and r respectively, and the dimensionless temperature T are defined by

$$r=r'/r_0\,,\quad x=x'/r_0\,\text{Pé}\,,\quad T=(T'-T_0)/(T_w-T_0)$$

where the prime refers to dimensional quantities, and T_0 is the constant and uniform temperature of the fluid at the tube inlet (assumed to be far upstream of the heated region). We shall consider the heated portion of the tube to have constant wall temperature T_w, but other boundary conditions can be treated in the manner to be considered here. This problem and the related problem of flow between heated parallel plates are usually called the Graetz problem; and, as indicated by Porter's review [1], it has been extensively studied since Graetz [2] examined it in 1885. Graetz neglected the axial conduction term, the last term in (1), but for low velocity flows and for liquid metals (systems for which $\text{Pé}<100$) the axial conduction cannot be neglected. Hsu [3, 4] obtained exact solutions for (1) for constant heat flux at the wall and for Newton's law of cooling boundary conditions in the heated region, but his analyses are not physically correct because he neglected the axial conduction across the plane $x=0$ (assumed here to be the inlet of the heated section). He decoupled the upstream ($x<0$) temperature field from the temperature field in the heated region ($x>0$) by assuming the temperature distribution at $x=0$ to be uniform at the inlet temperature T_0.

Schneider [5] and Agrawal [6] correctly formulated the problem, recognizing that axial conduction will distort the temperature distribution at $x=0$ if the Péclet number is small ($\text{Pé}<100$). Agrawal attempted to solve the problem for Poiseuille flow between parallel plates, but computational difficulties prevented him from satisfactorily matching the solutions at $x=0^+$ and $x=0^-$. Schneider, on the other hand, solved the special case of Newton's law of cooling in the upstream ($x<0$) and downstream ($x>0$) regions for plug flow (uniform velocity profile) in a circular tube and between parallel plates. Schneider's special case does not involve the difficulty of matching the solutions in the two domains that Agrawal encountered, but such simplification does not apply to other boundary conditions including the one we consider here.

It is the purpose of this paper to show that the Graetz problem with axial conduction can be solved by applying the generalized sampling theorem to relate the coefficients in the series expansion of the temperature fields in the two domains, thereby eliminating the need for approximations and numerical matching procedures. To illustrate the procedure we shall examine the case of plug flow in a circular tube perfectly insulated in the region $(x<0)$ and maintained at constant wall temperature T_w in the region $(x>0)$. The analysis can be extended to Poiseuille flow in tubes and between parallel plates and to other commonly encountered boundary conditions.

2. Analysis

For plug flow at the average velocity in the tube (1) reduces to

$$\frac{\partial T_i}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_i}{\partial r}\right) + \frac{1}{\text{Pé}^2}\frac{\partial^2 T_i}{\partial x^2}\,; \qquad i=1,2 \tag{2}$$

where in this case $\text{Pé}=\bar{u}r_0/\alpha$ and $\bar{u}$ is the average velocity. We shall consider the temperature fields T_1 and T_2 in the two domains $-\infty<x<0; 0<r<1$ and $0<x<\infty; 0<r<1$, respectively. The boundary conditions are

$$\lim_{x\to-\infty} T_1(x,r)=0 \tag{3}$$

$$\frac{\partial T_1}{\partial r}(x,1)=0 \tag{4}$$

$$\frac{\partial T_1}{\partial r}(x,0)=0 \tag{5}$$

$$\lim_{x\to\infty} T_2(x,r)=1 \tag{6}$$

$$T_2(x,1)=1 \tag{7}$$

$$\frac{\partial T_2}{\partial r}(x,0)=0\,. \tag{8}$$

The temperature fields must also satisfy the compatibility conditions

$$T_1(0,r)=T_2(0,r) \tag{9}$$

$$\frac{\partial T_1}{\partial x}(0,r)=\frac{\partial T_2}{\partial x}(0,r)\,. \tag{10}$$

Problems involving two or more temperature fields coupled through compatibility conditions at a mutual boundary have been called conjugated boundary value problems. We shall solve the conjugated boundary value problem described by (2) to (10) by applying the finite Hankel transform to obtain series solutions for the temperature fields $T_1(x,r)$ and $T_2(x,r)$. Then we apply the compatibility conditions, using the generalized sampling theorem as it was applied by Jerri [7, 8] to obtain relations among the coefficients of the eigenfunction expansions $T_1(0,r)$ and $T_2(0,r)$ to solve for these coefficients.

For the region $x>0$ let us apply the finite J_0-Hankel transform pair

$$\hat{f}(\lambda_{0,n})=\int_0^1 rJ_0(\lambda_{0,n}r)f(r)\,dr \tag{11}$$

$$f(r) = \sum_{n=1}^{\infty} \frac{2\hat{f}(\lambda_{0,n}) J_0(\lambda_{0,n} r)}{J_1^2(\lambda_{0,n})} \tag{12}$$

where $\lambda_{0,n}$ is chosen such that condition (7) is satisfied, *i.e.* $J_0(\lambda_{0,n})=0, n=1,2\ldots$. Transforming (2) and using (7) we obtain

$$\frac{1}{\text{Pé}^2}\frac{d^2\hat{T}_2}{dx^2} - \frac{d\hat{T}_2}{dx} - \lambda_{0,n}^2 \hat{T}_2 = -\lambda_{0,n} J_1(\lambda_{0,n}), \quad \text{for } x>0. \tag{13}$$

To solve (13) we note that condition (6) transforms to

$$\lim_{x\to\infty} \hat{T}_2(x, \lambda_{0,n}) = \frac{J_1(\lambda_{0,n})}{\lambda_{0,n}}. \tag{14}$$

Hence the solution to (13) is

$$\hat{T}_2(x, \lambda_{0,n}) = \frac{J_1(\lambda_{0,n})}{\lambda_{0,n}} + C(\lambda_{0,n}) \exp\left\{\frac{x\,\text{Pé}^2}{2}[1-B(\lambda_{0,n})]\right\} \tag{15}$$

where

$$B(\lambda_{0,n}) = \left(1 + \frac{4\lambda_{0,n}^2}{\text{Pé}^2}\right)^{\frac{1}{2}}.$$

The inverse of (15), using (12), is

$$T_2(x, r) = 1 + \sum_{n=1}^{\infty} \frac{2C(\lambda_{0,n}) J_0(\lambda_{0,n} r)}{J_1^2(\lambda_{0,n})} \exp\left\{\frac{x\,\text{Pé}^2}{2}[1-B(\lambda_{0,n})]\right\}, \; x>0. \tag{16}$$

It is clear that (16) satisfies (2), (6), (7) and (8).

For the region $x<0$ we apply the transform pair

$$\hat{f}(\lambda_{1,n}) = \int_0^1 r J_0(\lambda_{1,n} r) f(r) dr \tag{17}$$

$$f(r) = \sum_{n=1}^{\infty} \frac{2\hat{f}(\lambda_{1,n}) J_0(\lambda_{1,n} r)}{J_0^2(\lambda_{1,n})} \tag{18}$$

where $J_1(\lambda_{1,n})=0$ to satisfy condition (4). Thus (2) transforms to

$$\frac{1}{\text{Pé}^2}\frac{d^2\hat{T}_1}{dx^2} - \frac{d\hat{T}_1}{dx} - \lambda_{1,n}^2 \hat{T}_1 = 0, \qquad x<0 \tag{19}$$

where we have used condition (4). To solve (19) we again note that (3) transforms to

$$\lim_{x\to-\infty} \hat{T}_1(x, \lambda_{1,n}) = 0. \tag{20}$$

Hence the solution to (19) is

$$\hat{T}_1(x, \lambda_{1,n}) = D(\lambda_{1,n}) \exp\left\{\frac{x\,\text{Pé}^2}{2}[1+B(\lambda_1, n)]\right\} \tag{21}$$

where

$$B(\lambda_{1,n}) = \left(1 + \frac{4\lambda_{1,n}^2}{\text{Pé}^2}\right)^{\frac{1}{2}}.$$

Using the inversion formula (18) we obtain

$$T_1(x, r) = \sum_{n=1}^{\infty} \frac{2D(\lambda_{1,n}) J_0(\lambda_{1,n} r)}{J_0^2(\lambda_{1,n})} \exp\left\{\frac{x\,\text{Pé}^2}{2}[1+B(\lambda_{1,n})]\right\}, \; x<0. \tag{22}$$

Equation (22) satisfies (2), (3), (4) and (5). It remains to determine the coefficients $C(\lambda_{0,n})$ and

$D(\lambda_{1,n})$ in (16) and (22), respectively. To this purpose we use the compatibility conditions (9) and (10) together with the following sampling theorem, which will aid considerably in finding relations between the coefficients $C(\lambda_{0,n})$ and $D(\lambda_{1,n})$.

3. The sampling theorem

Given

$$\hat{f}(\lambda) = \int_I \rho(y) K(y, \lambda) f(y) dy \tag{23}$$

where $\{K(y, \lambda_n)\}$ is a complete orthogonal set on the finite interval I, and $f(y)$ is square integrable then

$$\hat{f}(\lambda) = \sum_{n=1}^{\infty} \hat{f}(\lambda_n) S(\lambda, \lambda_n) \tag{24}$$

where the sampling function is defined by

$$S(\lambda, \lambda_n) = \frac{\int_I \rho(y) K(y, \lambda) \overline{K(y, \lambda_n)} dy}{\int_I \rho(y) |K(y, \lambda_n)|^2 dy}. \tag{25}$$

The proof is readily established when we write the orthogonal expansion for $f(y)$ in (23),

$$f(y) = \sum_{n=1}^{\infty} C_n \overline{K(y, \lambda_n)},$$

$$C_n = \frac{\int_I \rho(y) f(y) K(y, \lambda_n) dy}{\int_I \rho(y) |K(y, \lambda_n)|^2 dy} = \frac{\hat{f}(\lambda_n)}{\int_I \rho(y) |K(y, \lambda_n)|^2 dy}$$

then multiply both sides by $\rho(y) K(y, \lambda)$ and integrate to obtain

$$\int_I \rho(y) K(y, \lambda) f(y) dy = \hat{f}(\lambda) = \sum_{n=1}^{\infty} \hat{f}(\lambda_n) S(\lambda, \lambda_n)$$

after using (23) for $\hat{f}(\lambda_n)$ and (24) for $S(\lambda, \lambda_n)$.

4. Application of the sampling theorem

For the special cases of interest here, (11) and (17), the sampling functions are

$$S_1(\lambda, \lambda_{0,n}) = \frac{2\lambda_{0,n} J_0(\lambda)}{(\lambda_{0,n}^2 - \lambda^2) J_1(\lambda_{0,n})} \tag{26}$$

and

$$S_2(\lambda, \lambda_{1,n}) = \frac{2\lambda J_1(\lambda)}{(\lambda^2 - \lambda_{1,n}^2) J_0(\lambda_{1,n})}, \tag{27}$$

respectively. We note that $S_1(\lambda_{0,m}, \lambda_{0,n}) = \delta_{m,n}$ and $S_2(\lambda_{1,m}, \lambda_{1,n}) = \delta_{m,n}$. Now recognizing that the coefficients $C(\lambda_{0,n})$ and $D(\lambda_{1,n})$ have the form of (23), we can write

$$C(\lambda) = \sum_{n=1}^{\infty} C(\lambda_{0,n}) S_1(\lambda, \lambda_{0,n}) \tag{28}$$

and

$$D(\lambda) = \sum_{n=1}^{\infty} D(\lambda_{1,n}) S_2(\lambda, \lambda_{1,n}). \tag{29}$$

Applying compatibility condition (9) to $T_2(0, r)$ and $T_1(0, r)$ obtained from (16) and (22), respectively, we have

$$\sum_{n=1}^{\infty} \frac{2D(\lambda_{1,n}) J_0(\lambda_{1,n} r)}{J_0^2(\lambda_{1,n})} = 1 + \sum_{n=1}^{\infty} \frac{2C(\lambda_{0,n}) J_0(\lambda_{0,n} r)}{J_1^2(\lambda_{0,n})}. \tag{30}$$

Multiplying both sides of (30) by $r J_0(\lambda r) dr$ and integrating term by term from 0 to 1 we obtain

$$\sum_{n=1}^{\infty} D(\lambda_{1,n}) S_2(\lambda, \lambda_{1,n}) = \frac{J_1(\lambda)}{\lambda} + \sum_{n=1}^{\infty} C(\lambda_{0,n}) S_1(\lambda, \lambda_{0,n}) \tag{31}$$

which, after employing (28) and (29), becomes

$$D(\lambda) - C(\lambda) = \frac{J_1(\lambda)}{\lambda}. \tag{32}$$

Next we apply compatibility condition (10) to give

$$\sum_{n=1}^{\infty} \frac{2D(\lambda_{1,n}) [1 + B(\lambda_{1,n})] J_0(\lambda_{1,n} r)}{J_0^2(\lambda_{1,n})} = \\ = \sum_{n=1}^{\infty} \frac{2C(\lambda_{0,n}) [1 - B(\lambda_{0,n})] J_0(\lambda_{0,n} r)}{J_0^2(\lambda_{0,n})}. \tag{33}$$

Multiplying both sides by $r J_0(\lambda r) dr$ and integrating as before we obtain

$$D(\lambda) + \sum_{n=1}^{\infty} D(\lambda_{1,n}) B(\lambda_{1,n}) S_2(\lambda, \lambda_{1,n}) \\ = C(\lambda) - \sum_{n=1}^{\infty} C(\lambda_{0,n}) B(\lambda_{0,n}) S_1(\lambda, \lambda_{0,n}) \tag{34}$$

where we have applied the sampling theorem to the first terms in the square brackets in (33) but not to the terms involving $B(\lambda_{1,n})$ and $B(\lambda_{0,n})$. Applying (32) after rearranging (34) we obtain

$$-\sum_{n=1}^{\infty} C(\lambda_{0,n}) B(\lambda_{0,n}) S_1(\lambda, \lambda_{0,n}) = \sum_{n=1}^{\infty} D(\lambda_{1,n}) B(\lambda_{1,n}) S_2(\lambda, \lambda_{1,n}) + \frac{J_1(\lambda)}{\lambda}. \tag{35}$$

To solve for $C(\lambda_{0,n})$ we let $\lambda = \lambda_{0,m}$ in (35), noting that $S_1(\lambda_{0,m}, \lambda_{0,n}) = \delta_{m,n}$. Thus

$$-C(\lambda_{0,m}) B(\lambda_{0,m}) = \sum_{n=1}^{\infty} D(\lambda_{1,n}) B(\lambda_{1,n}) S_2(\lambda_{0,m}, \lambda_{1,n}) + \frac{J_1(\lambda_{0,m})}{\lambda_{0,m}}. \tag{36}$$

To eliminate $D(\lambda_{1,n})$ in (36) we use (32) with $\lambda = \lambda_{1,n}$ to express $C(\lambda_{0,m})$ in terms of $C(\lambda_{1,n})$. The result is

$$-C(\lambda_{0,m}) B(\lambda_{0,m}) = \\ = \sum_{n=1}^{\infty} \left[C(\lambda_{1,n}) + \frac{J_1(\lambda_{1,n})}{\lambda_{1,n}} \right] B(\lambda_{1,n}) S_2(\lambda_{0,m}, \lambda_{1,n}) + \frac{J_1(\lambda_{0,m})}{\lambda_{0,m}}. \tag{37}$$

Since $\lambda_{1,1} = 0$ we can write

$$\frac{J_1(\lambda_{1,n})}{\lambda_{1,n}} = \begin{cases} \frac{1}{2} & \text{for} \quad n = 1 \\ 0 & \text{for} \quad n \neq 1 \end{cases}$$

and

$$S_2(\lambda_{0,m}, \lambda_{1,1}) = \frac{2J_1(\lambda_{0,m})}{\lambda_{0,m}}.$$

Using (28) with $\lambda = \lambda_{1,n}$ we obtain a relation between $C(\lambda_{1,n})$ and $C(\lambda_{0,j})$ as follows

$$C(\lambda_{1,n}) = \sum_{j=1}^{\infty} C(\lambda_{0,j}) S_1(\lambda_{1,n}, \lambda_{0,j}) \tag{38}$$

and since $\lambda_{1,1}=0$ we have

$$S_1(\lambda_{1,1}, \lambda_{0,m}) = \frac{2}{\lambda_{0,m} J_1(\lambda_{0,m})}.$$

Eliminating $C(\lambda_{1,n})$ from (37) using (38) we obtain

$$-C(\lambda_{0,m}) B(\lambda_{0,m}) =$$

$$= \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} C(\lambda_{0,j}) S_1(\lambda_{1,n}, \lambda_{0,j}) \; B(\lambda_{1,n}) S_2(\lambda_{0,m}, \lambda_{1,n}) + \frac{2J_1(\lambda_{0,m})}{\lambda_{0,m}}. \tag{39}$$

Interchanging the order of the summations and defining

$$K(\lambda_{0,m}, \lambda_{0,j}) \equiv \sum_{n=1}^{\infty} B(\lambda_{1,n}) S_2(\lambda_{0,m}, \lambda_{1,n}) S_1(\lambda_{1,n}, \lambda_{0,j}) \tag{40}$$

then (39) reduces to

$$-C(\lambda_{0,m}) B(\lambda_{0,m}) = \sum_{j=1}^{\infty} K(\lambda_{0,m}, \lambda_{0,j}) C(\lambda_{0,j}) + \frac{2J_1(\lambda_{0,m})}{\lambda_{0,m}}. \tag{41}$$

Now (41) can be solved to obtain the coefficients $C(\lambda_{0,m})$, and the coefficients $D(\lambda_{1,m})$ are calculated by applying (38) and (32).

5. Results

Once the coefficients $C(\lambda_{0,m})$ and $D(\lambda_{1,m})$ are known the temperature fields are obtained from (16) and (22). The coefficients $C(\lambda_{0,m})$ have been obtained by writing the infinite sequence of equations for $C(\lambda_{0,1})$, $C(\lambda_{0,2})$, ... from (41) as a matrix equation, truncating the system and solving the resulting matrix equation by a matrix inversion routine. The matrix equation to be solved is $AZ=Y$ where Z and Y are column matrices defined by $Z \equiv \{z_1 z_2 \ldots z_n\}$ with $z_i \equiv C(\lambda_{0,i})$ and $Y \equiv \{y_1 y_2 \ldots y_n\}$ with

$$y_i \equiv -\frac{2J_1(\lambda_{0,i})}{\lambda_{0,i}},$$

respectively. The matrix $A \equiv (a_{ij})$ has the coefficients given by $a_{ij} = K(\lambda_{0,i}, \lambda_{0,j}) + \delta_{i,j} B(\lambda_{0,i})$.

Numerical computations have been performed for several Péclet numbers, and typical results are presented in Table 1 and in Figures 1, 2 and 3. The coefficients $C(\lambda_{0,m})$, which are

TABLE 1

The coefficients $C(\lambda_{0,m})$ for various Péclet numbers

m	Pé = 1	Pé = 3	Pé = 5	Pé = 10
1	−0.0587876	−0.132404	−0.168412	−0.199655
2	0.0097031	0.023382	0.032488	0.046058
3	−0.0037665	−0.009135	−0.012793	−0.019164
4	0.0019858	0.004827	0.006733	0.010258
5	−0.0012263	−0.002989	−0.004134	−0.006341
6	0.0008342	0.002041	0.002801	0.004300
7	−0.0006060	−0.001491	−0.002024	−0.003111
8	0.0004618	0.001155	0.001533	0.002361
9	−0.0003649	−0.000914	−0.001204	−0.001858
10	0.0002969	0.000753	0.000973	0.001506

required to determine the temperature distribution in the heated region, are tabulated in Table 1 for various Péclet numbers.

Figures 1, 2 and 3 show temperature profiles at various axial positions for Pé = 1, 5 and 10, respectively. For Pé > 100 there is no appreciable effect of axial conduction, but the effects are very significant for Pé < 10. The solutions for the upstream ($x<0$) and downstream ($x>0$) temperature fields as $x \to 0$ are shown to be in excellent agreement as indicated in the figures by the temperature profiles for $x = \pm 0.00001$. The solutions shown involved the use of twenty

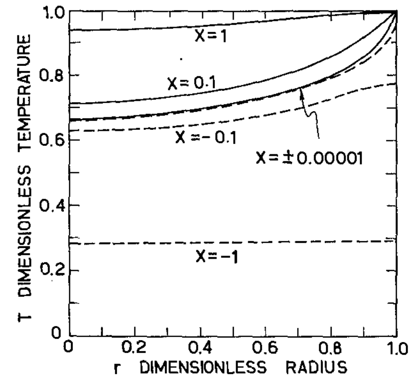

Figure 1. Temperature profiles as a function of axial position for Pé = 1. ---- from Equation (22), —— from Equation (16).

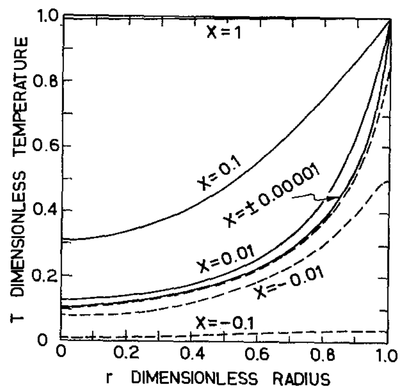

Figure 2. Temperature profiles as a function of axial position for Pé = 5. ---- from Equation (22), —— from Equation (16).

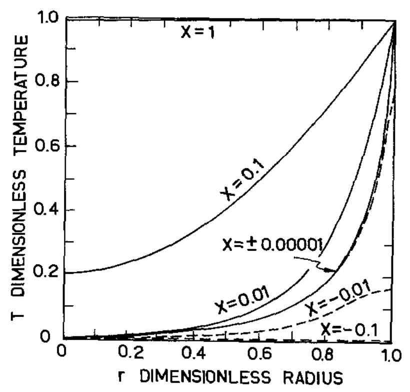

Figure 3. Temperature profiles as a function of axial position for Pé = 10. ---- from Equation (22), —— from Equation (16).

terms in each series expansion for the temperature fields, but fifteen terms are satisfactory for most purposes.

For Pé = 10 the temperature field near the wall is seen to be distorted in the upstream region due to axial conduction. The asymptotic solution in the upstream region is attained for $|x| \approx 0.1$, but in the downstream region the asymptotic temperature distribution is reached for $|x| > 1$. As the Péclet number is decreased the effects of axial conduction become more pronounced, and for Pé = 1 the temperature field in the upstream region is affected for $|x| > 1$. It is to be expected that for sufficiently small Péclet numbers the axial conduction predominates over the convective transport of energy, and in the limit as Pé→0 ($\bar{u}\to 0$) the governing equation reduces to the axisymmetrical Laplace equation in cylindrical coordinates

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'}{\partial r'}\right) + \frac{\partial^2 T'}{\partial x'^2} = 0 \tag{42}$$

where the primes refer to dimensional quantities. In the limit as Pé→∞ axial convection predominates over axial conduction, and the axisymmetrical conduction equation

$$\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{43}$$

is recovered.

We have treated the model of plug flow to illustrate the application of the sampling theorem, but the method can be applied to Poiseuille flow in tubes and between parallel plates. The physical interpretation provided here should extend to more realistic flows, in fact, the effect of axial conduction must be more pronounced for flows in which the velocity vanishes at the wall because axial convection is then reduced in the vicinity of the wall. This suggests that Hsu's assumption that the boundary condition at $x=0$ can be assumed *a priori* to be the constant inlet temperature is not physically correct nor mathematically justifiable.

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